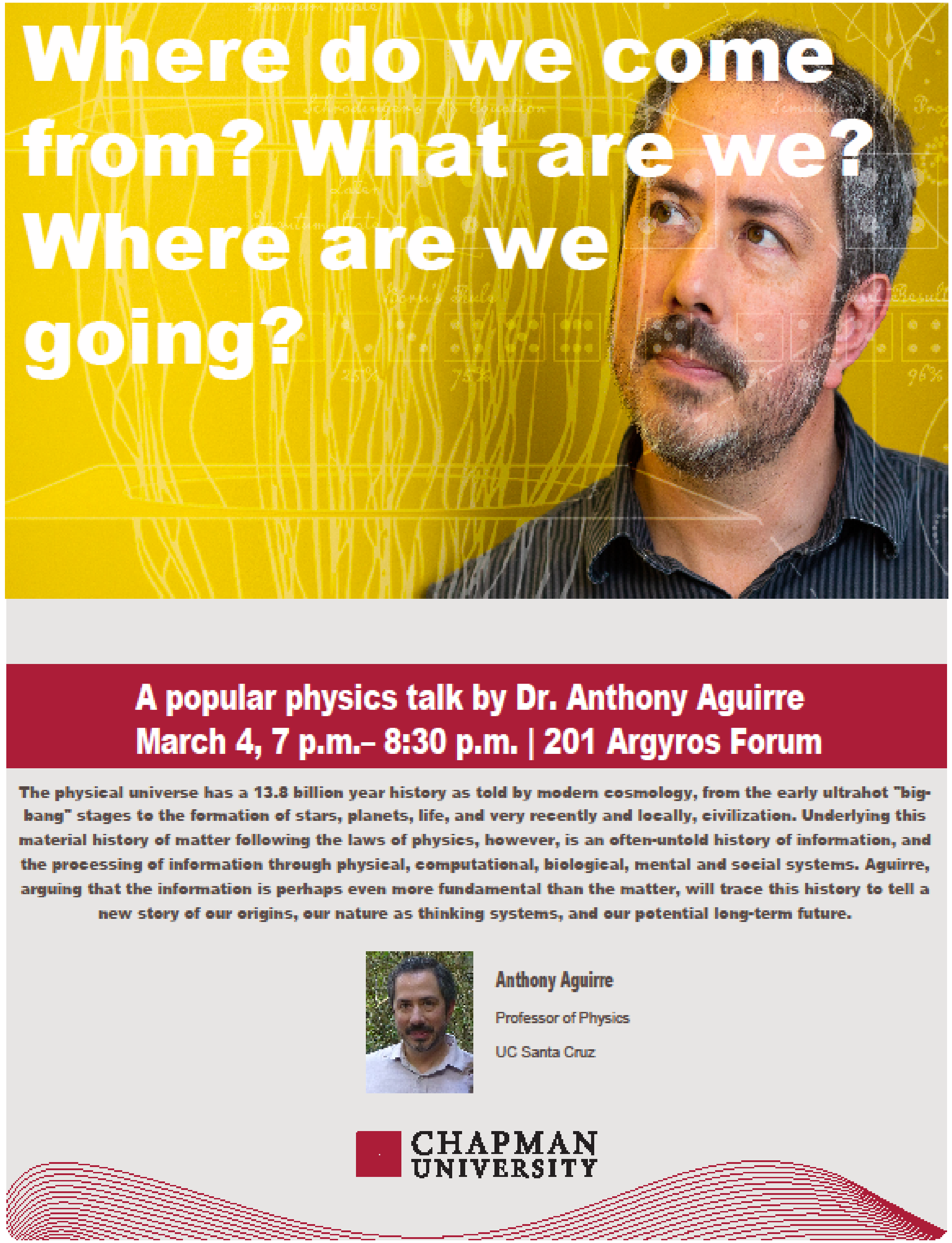
PHYS/PHIL 329 Lecture 6: The two-path experiments

Kelvin McQueen - 2/20/2020

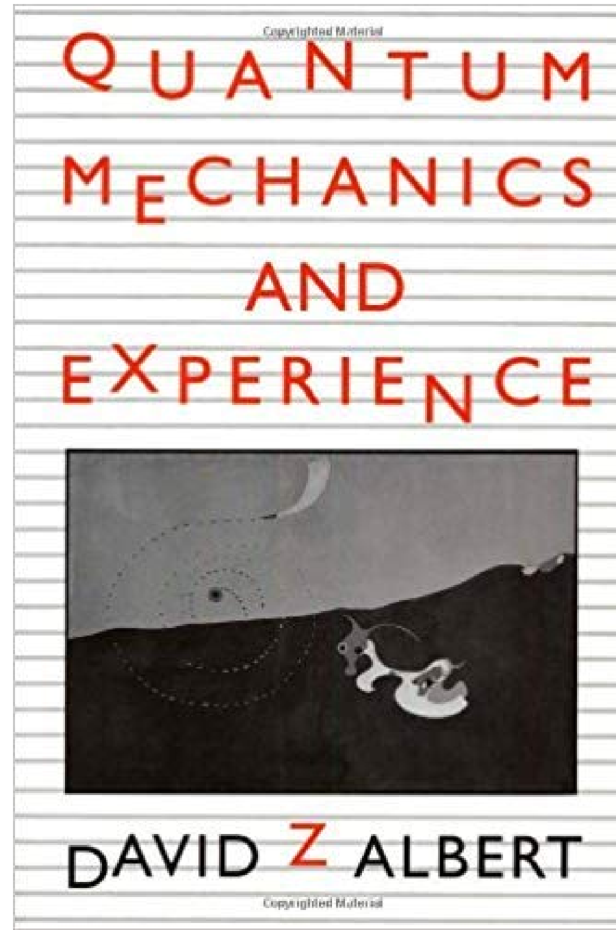
Announcements

- Problem Set 1 has been posted on Canvas. Due date is March 3, 11:59pm
  - You only have to complete 5 out of 10 assignments, but you must include at least one problem set and one essay.
  - We recommend that everyone submit problem set 1.
  - Please come to office hours if you are having difficulty. You may ask questions about the problem set while you are working on it and before you have submitted it.
    - Leifer: 10am-11am Mondays and Fridays
    - McQueen: 4pm-5pm Tuesdays and Thursdays
    - https://calendly.com/leifer to book an appointment with Dr. Leifer outside office hours.
  - Solutions must be submitted online on Canvas. Use an app like Microsoft Office Lens to scan hand-written solutions


Where do we come from? What are we? Where are we going?
A popular physics talk by Dr. Anthony Aguirre
March 4, 7 p.m.– 8:30 p.m. | 201 Argyros Forum
The physical universe has a 13.8 billion year history as told by modern cosmology, from the early ultrahot "big-bang" stages to the formation of stars, planets, life, and very recently and locally, civilization. Underlying this material history of matter following the laws of physics, however, is an often-untold history of information, and the processing of information through physical, computational, biological, mental and social systems. Aguirre, arguing that the information is perhaps even more fundamental than the matter, will trace this history to tell a new story of our origins, our nature as thinking systems, and our potential long-term future.
Anthony Aguirre
Professor of Physics
UC Santa Cruz
CHAPMAN UNIVERSITY

Today's Lecture

- Albert's discussion of the two path experiments (p53-57).
- Does quantum mechanics explain/describe reality?
- The Copenhagen interpretation of quantum mechanics.



Recap: the position operator X

- Eigenvalues of the X operator are points along the *x*-dimension of physical (coordinate) space:
  - $X|X = 5\rangle = 5|X = 5\rangle$
- Eigenvectors of any property operator, which have distinct eigenvalues, must be *orthogonal*. E.g. consider color…
  - $(\langle white|black\rangle)^2 = (\cos(90))^2 = 0^2 = 0.$
    - [A white electron is never found to be black, and vice versa.]
  - $(\langle white|white\rangle)^2 = (\cos(0))^2 = 1^2 = 1.$
    - [Color measurements are repeatable.]
- Hence, the vector space that represents the possible *positions* of a particle must be *infinitely dimensional*.
- This leads to the more general notion of a *wavefunction*.
- But for our purposes it is easy to treat space as containing only those points that interest us…

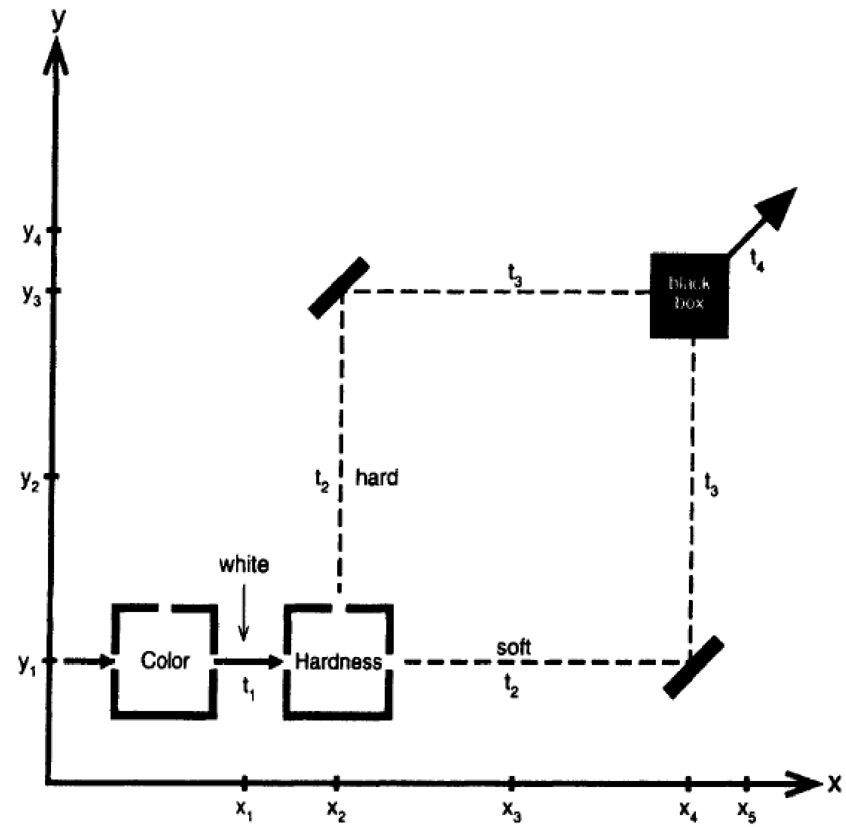
Recap: representing particle position

- To describe the two-path experiments, Albert pretends that space only has six locations:

  (x_1,y_1), (x_2,y_2), (x_3,y_1),
  (x_3,y_3), (x_4,y_2), (x_5,y_4).

- This gives a 6D vector space whose basis vectors can be written down as:

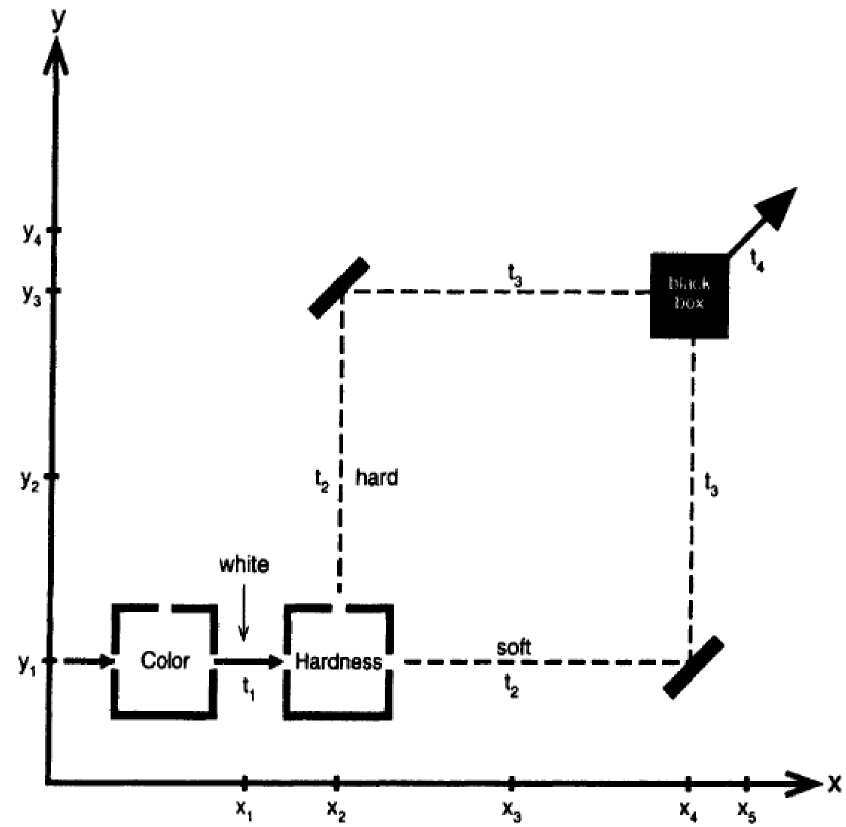
  $|X = x_1, Y = y_1\rangle$
  $|X = x_2, Y = y_2\rangle$
  $|X = x_3, Y = y_1\rangle$
  $|X = x_3, Y = y_3\rangle$
  $|X = x_4, Y = y_2\rangle$
  $|X = x_5, Y = y_4\rangle$



Recap: Representing position and color/hardness

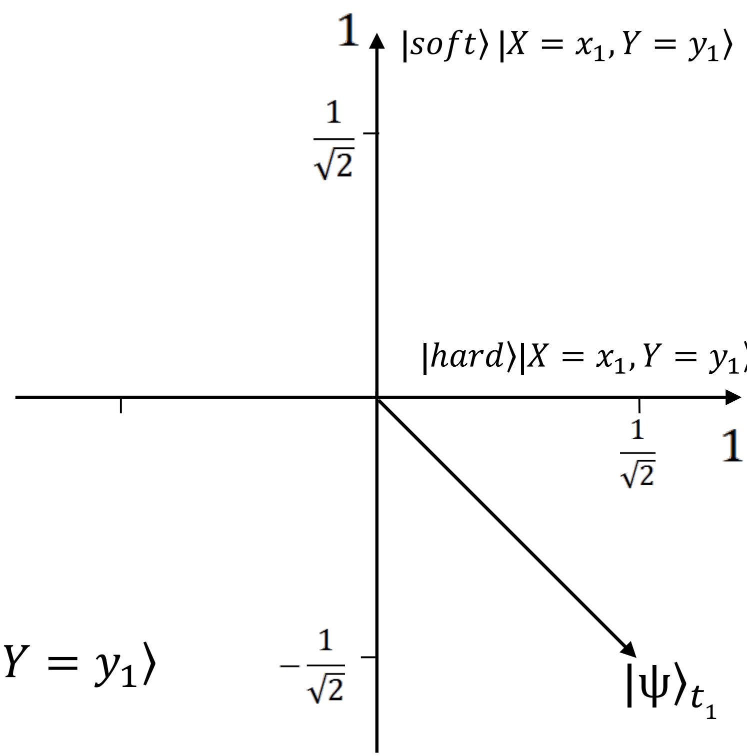
- To model our experiments we must also represent spin space properties (color, hardness, etc.)
- We combine the 6D vector space with our familiar 2D vector space into a 6x2=12D vector space, whose basis vectors can be written as:

  $|hard\rangle|X = x_1, Y = y_1\rangle$
  $|soft\rangle\,|X = x_1, Y = y_1\rangle$
  $|hard\rangle|X = x_2, Y = y_2\rangle$
  $|soft\rangle\,|X = x_2, Y = y_2\rangle$
  $|hard\rangle|X = x_3, Y = y_1\rangle$
  $|soft\rangle\,|X = x_3, Y = y_1\rangle$
  etc.



Recap: *separability*

- Let's represent a 2D subspace of our 12D vector space geometrically, using the first two basis vectors from our list.
- Since the position terms are identical, all (unit-length) vectors in this subspace represent states who spin-space and coordinate space values are *separable*.
  - For example consider $|\psi\rangle_{t_1}$



$$|\psi\rangle_{t_1} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_1, Y = y_1\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_1, Y = y_1\rangle$$

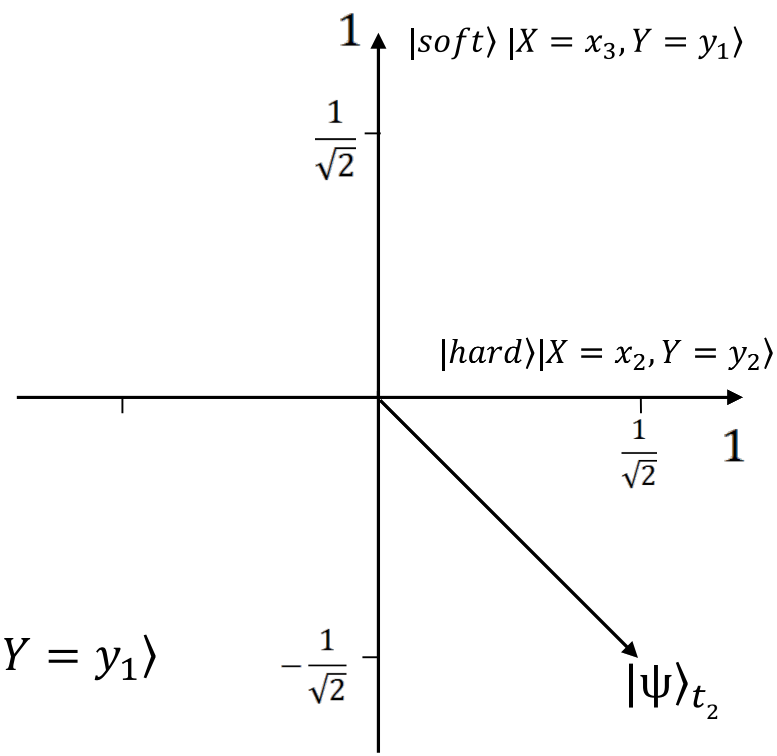
$$= \left(\frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle\right)|X = x_1, Y = y_1\rangle$$

$$= |white\rangle|X = x_1, Y = y_1\rangle$$

The possibility of the 2nd form of the expression means the particle's *position* and *hardness* are *separable*.

Recap: *nonseparability*

- If we instead choose a 2D subspace whose basis vectors involve different values for position (and hardness), then we can easily find vectors that represent *nonseparable* states.
  - For example consider $|\psi\rangle_{t_2}$



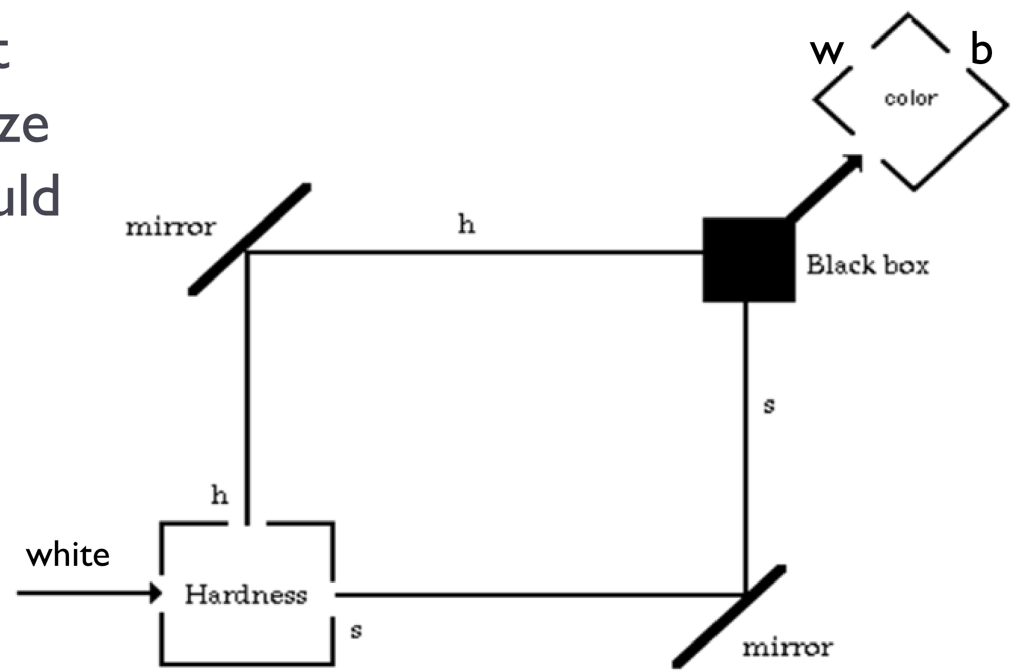
$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_2, Y = y_2\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_3, Y = y_1\rangle$$

- This state cannot be separated into definite color and/or position states.
- In particular:

$$|\psi\rangle_{t_2} \neq |white\rangle\left(\frac{1}{\sqrt{2}}|X = x_2, Y = y_2\rangle - \frac{1}{\sqrt{2}}|X = x_3, Y = y_1\rangle\right)$$

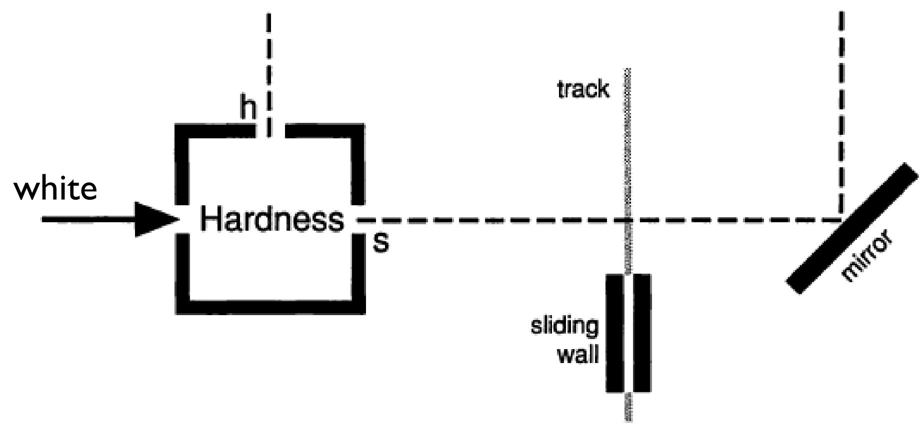
Recap: 2-path experiment 3

- Send white electrons through and then measure their color.
- Expected result:
  - 50% white 50% black.
  - Reason: we've learnt that hardness boxes randomize color, the color box should confirm this.
- Actual result:
  - 100% white!



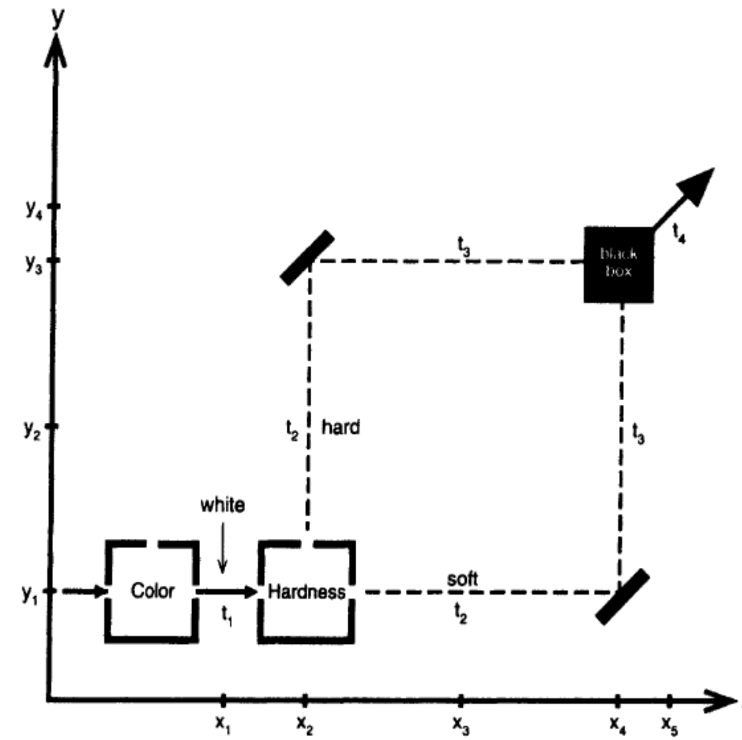
Recap: 2-path experiment 4

- Repeat experiment 3 (i.e. send white electrons through and then measure their color) but insert a stopping wall on the s-path.
- Expected result:
  - 50% less electrons, 100% will be white.
  - Reason: same experiment as 3, but we are blocking half the electrons?
- Actual result:
  - 50% less electrons, 50% white, 50% black.
  - What is going on!?



2-path experiment 3: t_1

- At time t_1, the electron has exited the white aperture of the color box and is about to go into the hardness box.



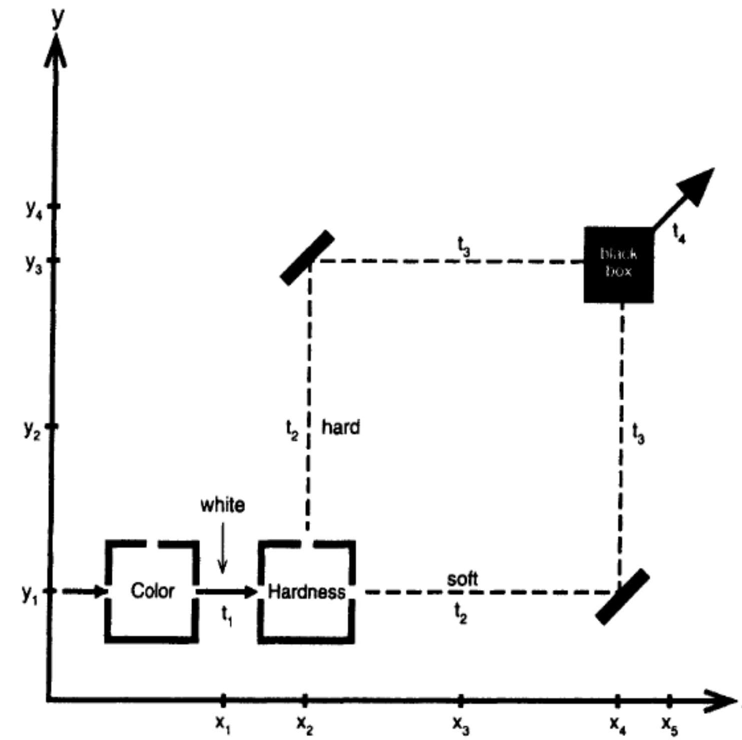
$$|\psi\rangle_{t_1} = |white\rangle|X = x_1, Y = y_1\rangle$$

$$= \left(\frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle\right)|X = x_1, Y = y_1\rangle$$

$$= \frac{1}{\sqrt{2}}|hard\rangle|X = x_1, Y = y_1\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_1, Y = y_1\rangle$$

2-path experiment 3: t_2

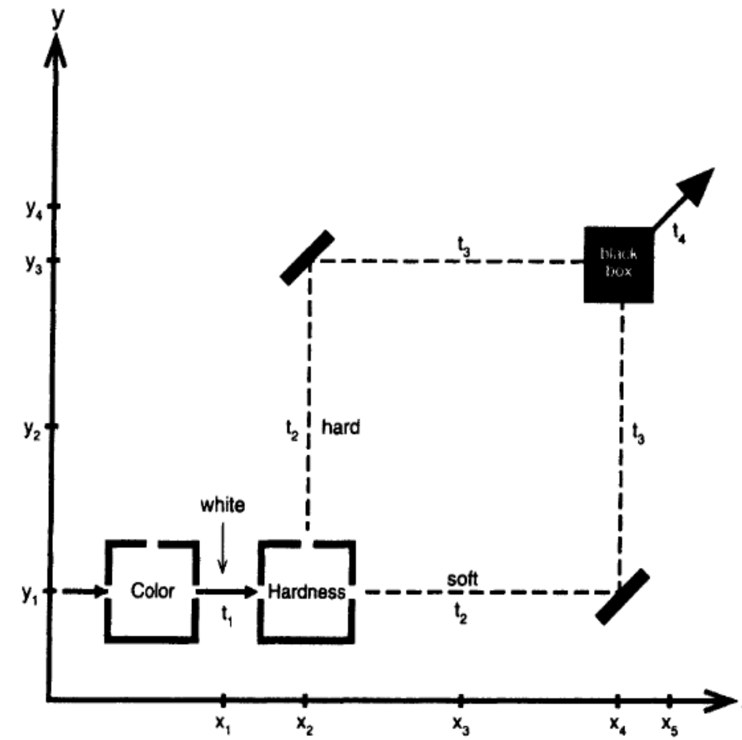
- At time t_2, the electron has exited the hardness box and is (presumably) heading towards the mirror(s).
- To determine the t_2 state, we can use *the linearity of the dynamics*.
- Thus, take the expanded t_1 state:
  - $|\psi\rangle_{t_1} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_1, Y = y_1\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_1, Y = y_1\rangle$
- Now figure out where an electron would be if its initial state vector was just the first term:
  $|hard\rangle|X = x_1, Y = y_1\rangle \rightarrow |hard\rangle|X = x_2, Y = y_2\rangle$
- Then do the same for the second term:
  $|soft\rangle|X = x_1, Y = y_1\rangle \rightarrow |soft\rangle|X = x_3, Y = y_1\rangle$
- Now put them back together into the superposition:



$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_2, Y = y_2\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_3, Y = y_1\rangle$$

2-path experiment 3: t_3

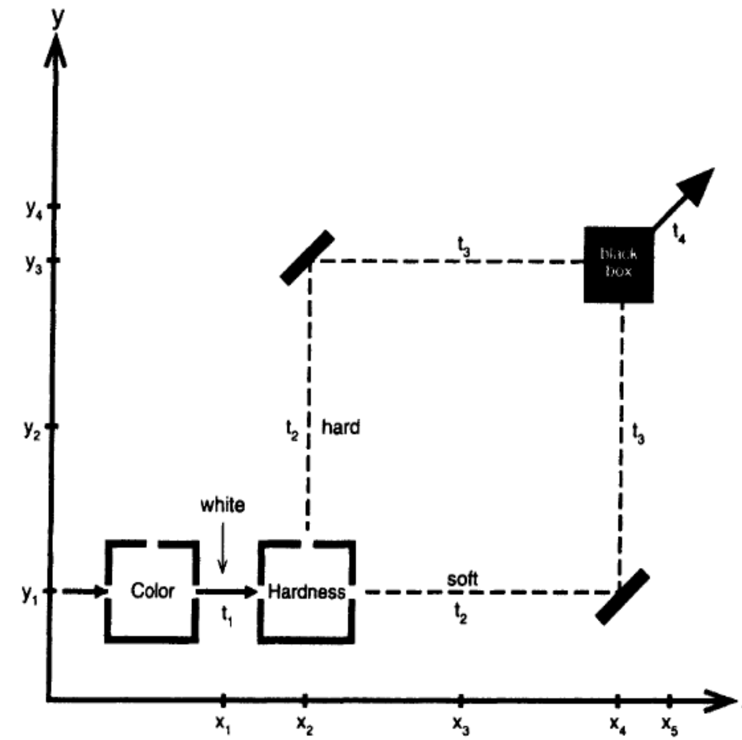
- At time t_3, the electron has (presumably) bounced off the mirror(s) and is heading towards the black box.
- To determine the t_3 state, we can use *the linearity of the dynamics* again.
- Notice that we still have a *nonseparable* state.



$$|\psi\rangle_{t_3} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_3, Y = y_3\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_4, Y = y_2\rangle$$

2-path experiment 3: t_4

- By t_4 the paths have been recombined giving:
  - $|\psi\rangle_{t_4} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_5, Y = y_4\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_5, Y = y_4\rangle$



- Notice the two position terms are identical.
- The state is now *separable*.
- So we can rewrite it a follows:

$$|\psi\rangle_{t_4} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_5, Y = y_4\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_5, Y = y_4\rangle$$

$$= \left(\frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle\right)|X = x_5, Y = y_4\rangle \qquad = |white\rangle|X = x_5, Y = y_4\rangle$$

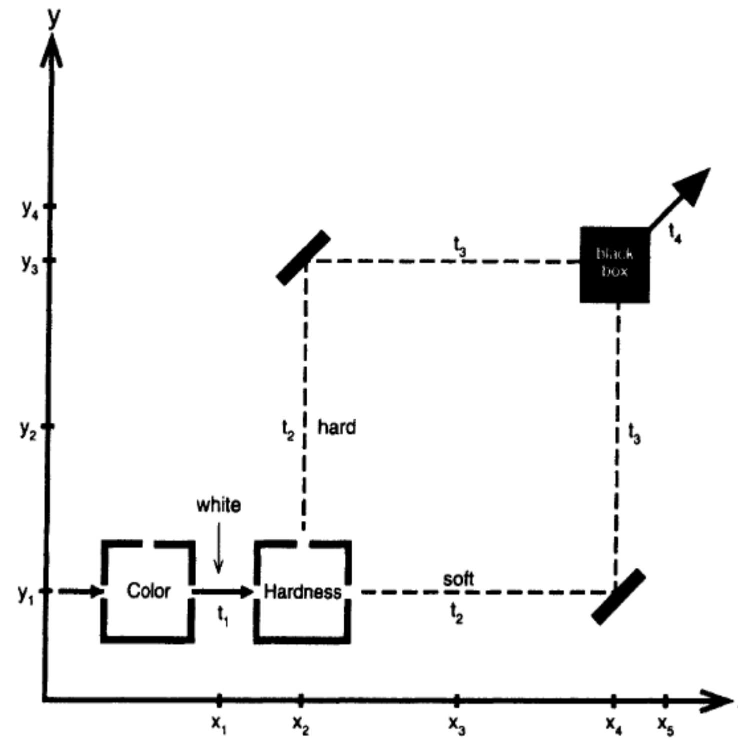
All white electrons!

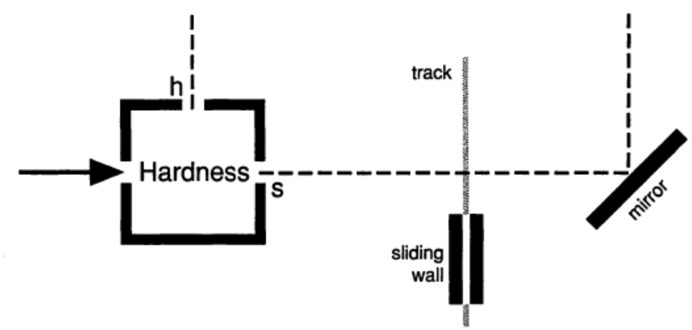
2-path experiment 4: t_3

- What if we place a wall on the soft path?
- Then at time t_3…

- Instead of this state:
  - $|\psi\rangle_{t_3} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_3, Y = y_3\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_4, Y = y_2\rangle$

- We have this state:
  - $|\psi'\rangle_{t_3} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_3, Y = y_3\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_3, Y = y_1\rangle$





2-path experiment 4: t_4

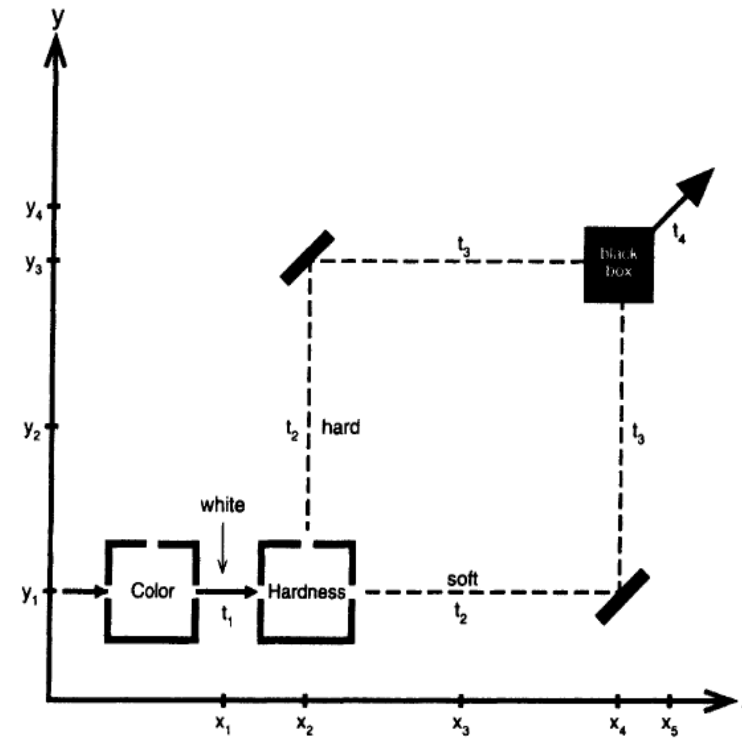
- Because of the wall, the state at t_4 remains *nonseparable*.
- Thus, a position measurement at $X = x_5, Y = y_4$, will find a particle to be there with probability:

  $$\langle \psi'_{t4} | hard, X = x_5, Y = y_4 \rangle = 0.5.$$

- If found, the electron's hardness is guaranteed to be:

  $$|hard\rangle = \frac{1}{\sqrt{2}}|black\rangle + \frac{1}{\sqrt{2}}|white\rangle$$

- Hence why a color measurement now gets 50/50 results!



$$|\psi'\rangle_{t_4} = \frac{1}{\sqrt{2}}|hard\rangle|X = x_5, Y = y_4\rangle - \frac{1}{\sqrt{2}}|soft\rangle|X = x_3, Y = y_1\rangle$$

Prediction, explanation, description

- Key questions: does quantum mechanics…
  - *predict* the results of the two-path experiments?
  - *explain* these results?
  - *describe* the actual properties of the particles as they traverse the paths, thereby enabling us to understand their objective structure?

The Copenhagen interpretation

- The Copenhagen interpretation is a possible solution to the measurement problem that comes from Neils Bohr.

- Albert describes it as follows:
  - "The right way to think about superpositions of, say, being black and being white is to think of them as situations wherein color talk is unintelligible. Talking and enquiring about the color of an electron in such circumstances is (on this view) like talking or inquiring about, say, whether the number 5 is still a bachelor. On this view the contradictions of chapter 1 go away because superpositions are situations wherein the superposed predicates just don't apply." (p38.)

- Measurement therefore brings the world into a state in which we can meaningfully talk about it!